**Video Study 1**

**“Thinking about Fractions: moving beyond one half and two quarters”**

**Video 1 Introduction**

In this video clip, two Grade 4 students are thinking about the fraction 2/5 (two fifths). The two students have explored simple fractions such as one half;

three thirds; two fourths or two quarters previously in Grade 3, but have not encountered this ‘unfamiliar’ fraction. In this preliminary task, the teacher planning team wanted to observe how students thought about unfamiliar fractions without prior direct instruction. The teachers considered this math task to be a diagnostic assessment opportunity that would reveal the misconceptions about fractions that students were working with in the fall of Grade 4 (age 9) in Ontario, Canada.

The task students were assigned was set out as follows:

* Work with a partner.
* Choose one fraction – either 2/5 or 1/10
* Represent the fraction in several ways using manipulatives/materials available on the table and using markers and poster paper.

*Now let’s watch the video!*

**Transcript**

St1: Okay so we, to figure out two fifths, we made rectangle with 3 lines vertical and 3 lines horizontal.

R: How many lines vertical?

St1: Um Five.

R: Okay why did you use 5 and 2 again?

St1: Because in two fifths, there’s a 2 and 5.

R: Okay

St1: So we did 5 two times and got ten.

R: Oh, so you found five and coloured it in?

St1: Yes

R: and then you found another 5 and coloured it in.

St2: and then we did uh two groups

St 1: The same

St 2: yea

St 2: But instead we coloured in two, five times. Like two squares, five times.

R: And what does that show?

St 1: That shows 10.

R: So you’ve got ten squares coloured in your…?

T: Ah, one, two, three, four five, six, seven, eight, nine, ten. Ah!

St 1: Cause we did, on this one we did five, two times I think. One two three four five.

T: So Eva does this have something to do with your last statement?

St 2: Yes.

T: So explain the last statement then.

St 2: Okay um I thought that two fifths might equal one tenth so we tried it with a chart. Because it does say two fifths.

St 1: So we are still trying to figure out if it is like multiplication, addition or division.

St 2: I think it is like addition because we have like, two fives.

St 2: No but two plus five equals seven, and two *times* five equals ten.

R: So can I just ask – would it be fair to say that this one is showing two fives and this one is showing one ten?

St 1: I guess.

R: Is it?

St 1: Yah.

**Video Guide Questions**

What assets are the students demonstrating in terms of their understanding of number sense?

What do the students seem to understand about fractions?

What do the students seem to be lacking in understanding about fractions?

How are fractions like division?

Do these students understand what two fifths means?

What would you do next, as the teacher, to help these students with their understanding of two fifths (and of fractions)?

**Possible Discussion Points**

*What assets are the students demonstrating in terms of their understanding of number sense and math abilities overall?*

* Use of math language related to operations is clear
* Ability to perform simple calculations such as 2 x 5
* Confidence and willingness to think about mathematics that they have not encountered previously
* Attempt made to find an equivalent fraction
* Use of an area model to represent the fraction

*What do the students seem to understand about fractions?*

* How to write a fraction with a numerator on top of line and denominator below the line
* Area models might help to represent a fraction
* That there is a relationship between the two and the five in two fifths

*What do the students seem to be lacking in understanding about fractions?*

*How are fractions like division?*

### Fraction notation is similar to division notation. Notice that the way we write fractions is similar to the way we usually note division: The division symbol is a dot above a horizontal line and a dot below the same line. These dots could be any numerals to give the fraction meaning.

### 2/5 is the same as 2 divided by 4: Let’s take 2 pieces of square paper. They are identical (the same shape, size, area, perimeter, etc). One is coloured blue and one is coloured orange.

### Now I am going to share the two squares of paper equally among five students.

### So I will divide each piece of paper into 5 equal portions.

### Each of these sections is called a fifth.

### Now each student gets a fair share of the paper strips. Each of the five students can have ONE blue strip and ONE orange strip. That means that each student gets two fifths.

For more information about this try watching: *http://www.youtube.com/watch?v=bbDRdQGJOLM*

*Do these students understand what two fifths means?*

* No, these students understand that there is a relationship between the two and the five and they are looking for that relationship. They also understand that they are related by an operation. The problem is, they don’t know see the numerator and denominator as representing one idea: two fifths rather than a two and a five.
* The students are unable to represent two fifths accurately

*What would you do next, as the teacher, to help these students with their understanding of two fifths (and of fractions)?*

Evaluate the following teacher moves. What might you do?

* **Option 1**

Explain the meaning of two fifths in context. The denominator tells how many parts there are in total and the numerator tells us how many parts we are focusing on. So let’s say you have a pan of brownies cut into five equal pieces. That shows the fifths. Now two of those cut pieces are for you. So you get two fifths of all the brownie in the pan.

* **Option 2**

Show the students a linear model of fractions. Start with a line or a strip of paper with a 0 at one end and a 1 at the other end. Tell the students to imagine walking to school from home along that line. Home is zero and school is one. Ask the students to move their fingers along the line until they get two fifths of the way to school. Then stop. How can they be sure that they have stopped at two fifths of the way along the line? Students might then want to partition the line into five equal parts (by folding for example). Once the students have found the two fifths point, mark it with fraction notation.

* **Option 3**

Have these two students work through Fractions CLIPS online learning objects at [www.mathclips.ca](http://www.mathclips.ca). See (Representing Simple Fractions activity list).

**Additional Commentary from invited expert Dr. Joan Moss**

This is a general diagnostic task that matches exactly with what we know from the literature (over 30 years of research), that children do not intuitively understand the meaning of the fraction symbol. In this case, the two girls do not understand the meaning of two over five. They treat the numbers as whole numbers – two and five. But they don’t understand that the line means ‘divided by’ even though they play around with that idea a bit.

The fraction symbol of 2 over 5 has little meaning to these students. They might very well be able to show what 40% means or one half. But the girls have not connected these understandings to the task at hand.

An additional puzzling issue here is that when the students do make an area model for the fraction, they likely had the idea to divide the rectangle into a 2 by 5 grid to represent the 2 and the 5. But for some reason, they show a 3 by 5 grid instead. When questioned about this, the girls do not self-correct. The students are focused on the manipulation of numbers but don’t demonstrate understanding of fractions either pictorially or numerically.

These students are missing what a fraction actually means.

But we do know that students CAN understand rational numbers and fractions. And these students clearly have the potential to learn quite a bit about fractions. The fraction symbol is complex. In my opinion, the way to support these students at this stage would be:

* Return to very common fractions and help students re-connect with their sense of what a fraction really means: For example talk about one quarter and the meanings of quarters (see lesson sequence at the Digital Video paper on Fractions for example of lessons that focus on key concepts related to fractions)
* Then focus on a quarter of something (as an operator).
* THEN return to 2/5ths using linear models such as a piece of string to help them connect into the meaning of this fraction.

Another question we might want to consider is this:

Are these students overly focused on this notion of showing their thinking in pictures, numbers and words? What might be a very important immediate next step for these students is to provide a structured opportunity to critically analyse their various responses for accuracy of meaning, and to revise their work.